Broken time-reversal-symmetry in triplet superconductor junctions

Philip M. R. Brydon\textsuperscript{1} C. Iniotakis\textsuperscript{2} Dirk Manske\textsuperscript{3} M. Sigrist\textsuperscript{2}

\textsuperscript{1}Institut für Theoretische Physik, Technische Universität Dresden
\textsuperscript{2}Institut für Theoretische Physik, ETH Zürich
\textsuperscript{3}Max-Planck-Institut für Festkörperforschung, Stuttgart

Nagoya, NDSN 2009, 4 September 2009
Spin of triplet Cooper pair is novel degree of freedom in Josephson junction physics.
Spin of triplet Cooper pair is **novel degree of freedom** in Josephson junction physics.

Already many new and interesting effects, e.g. spin current, unconventional interplay with magnetism.
 Spin of triplet Cooper pair is novel degree of freedom in Josephson junction physics.

 Already many new and interesting effects, e.g. spin current, unconventional interplay with magnetism.

 Spin pairing state of each superconductor can be set independently.
Spin of triplet Cooper pair is novel degree of freedom in Josephson junction physics.

Already many new and interesting effects, e.g. spin current, unconventional interplay with magnetism.

Spin pairing state of each superconductor can be set independently.

Reconstruction of tunneling Cooper pair spin state.
Spin of triplet Cooper pair is a novel degree of freedom in Josephson junction physics. Already many new and interesting effects, e.g. spin current, unconventional interplay with magnetism. Spin pairing state of each superconductor can be set independently. Reconstruction of tunneling Cooper pair spin state.

What are the consequences of different spin pairing states in the two superconductors?
The \( \mathbf{d} \)-vector

- gap is a spin matrix and written in terms of the \( \mathbf{d} \)-vector:

\[
\Delta(k) = \begin{bmatrix}
-d_x + id_y & d_z \\
 d_z & d_x + id_y
\end{bmatrix} = -i(\mathbf{d} \cdot \hat{\sigma})\hat{\sigma}_y
\]
The $d$-vector

- gap is a spin matrix and written in terms of the $d$-vector:

\[
\Delta(k) = \begin{pmatrix}
-d_x + id_y & d_z \\
d_z & d_x + id_y
\end{pmatrix}

= -i(d \cdot \hat{\sigma}) \hat{\sigma}_y

- the spin of the Cooper pair lies in the plane perpendicular to $d$
The \( \mathbf{d} \)-vector

- gap is a spin matrix and written in terms of the \( \mathbf{d} \)-vector:

\[
\Delta(\mathbf{k}) = \begin{bmatrix}
-d_x + id_y & d_z \\
d_z & d_x + id_y
\end{bmatrix}
\]

\[
= -i(\mathbf{d} \cdot \hat{\sigma})\hat{\sigma}_y
\]

- the spin of the Cooper pair lies in the plane perpendicular to \( \mathbf{d} \)

- unitary if \( \mathbf{d} \times \mathbf{d}^* = 0 \): condensate has no net spin
The $d$-vector

- gap is a spin matrix and written in terms of the $d$-vector:

$$\Delta(k) = \begin{bmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{bmatrix} = -i(d \cdot \hat{\sigma}) \hat{\sigma}_y$$

- the spin of the Cooper pair lies in the plane perpendicular to $d$
- unitary if $d \times d^* = 0$: condensate has no net spin
- equal spin pairing if $d$-vector points along same axis for all $k$
Misaligned $d$-vectors

- Josephson junction between superconductors with misaligned $d$-vectors
- Josephson spin current (Asano PRB 2005; Grønsleth et al., PRL 2006; PMRB, Manske and Sigrist, JPSJ 2008)

Broken time-reversal symmetry...
Josephson junction between superconductors with misaligned \( d \)-vectors

**Josephson spin current** (Asano PRB 2005; Grønsleth et al., PRL 2006; PMRB, Manske and Sigrist, JPSJ 2008)

mis-match of spin-pairing state produces effective spin

\[
S = id_L \times d_R^* + H.c.
\]

time-reversal-symmetry is broken
Josephson junction between superconductors with misaligned \( d \)-vectors

Josephson spin current (Asano PRB 2005; Grønsleth et al., PRL 2006; PMRB, Manske and Sigrist, JPSJ 2008)

mis-match of spin-pairing state produces effective spin

\[
S = id_L \times d_R^* + H.c.
\]

time-reversal-symmetry is broken

Can this induce a magnetization of the tunneling barrier?
Ginzburg-Landau Theory

Total free energy: magnetic and electronic contributions

\[ F = F_{\text{mag}} + F_{\text{el}} = \frac{|M|^2}{2\chi} + F_{\text{el}}(0) + \frac{\hbar}{2e} \int_0^\phi d\phi' I_J(\phi'), \quad \chi > 0 \]
Ginzburg-Landau Theory

Total free energy: magnetic and electronic contributions

\[ F = F_{\text{mag}} + F_{\text{el}} = \frac{|M|^2}{2\chi} + F_{\text{el}}(0) + \frac{\hbar}{2e} \int_0^\phi d\phi' I_J(\phi'), \quad \chi > 0 \]

Expand electronic free energy to lowest order in \( M \):

\[ F = \frac{|M|^2}{2\chi} - 2t d_L \cdot d_R \cos(\phi) + 2\gamma M \cdot (d_L \times d_R) \sin(\phi) \]
Ginzburg-Landau Theory

Total free energy: magnetic and electronic contributions

\[ F = F_{\text{mag}} + F_{\text{el}} = \frac{|M|^2}{2\chi} + F_{\text{el}}(0) + \frac{\hbar}{2e} \int_0^\phi d\phi' I_J(\phi'), \quad \chi > 0 \]

- Expand electronic free energy to lowest order in \( M \):

\[ F = \frac{|M|^2}{2\chi} - 2t d_L \cdot d_R \cos(\phi) + 2\gamma M \cdot (d_L \times d_R) \sin(\phi) \]

- “normal” \( M = \phi = 0 \) state unstable when

\[ \chi \gamma^2 |d_L \times d_R|^2 > t d_L \cdot d_R \]
Total free energy: magnetic and electronic contributions

\[ F = F_{\text{mag}} + F_{\text{el}} = \frac{|M|^2}{2\chi} + F_{\text{el}}(0) + \frac{\hbar}{2e} \int_0^\phi d\phi' I_J(\phi'), \quad \chi > 0 \]

Expand electronic free energy to lowest order in \( M \):

\[ F = \frac{|M|^2}{2\chi} - 2t d_L \cdot d_R \cos(\phi) + 2\gamma M \cdot (d_L \times d_R) \sin(\phi) \]

“normal” \( M = \phi = 0 \) state unstable when
\[ \chi \gamma^2 |d_L \times d_R|^2 > t d_L \cdot d_R \]

What determines \( \gamma \) and \( t \)?
Localized states with sub-gap energies form at the junction interface $\Rightarrow$ Andreev bound states $E_{p,k}$
Localized states with sub-gap energies form at the junction interface ⇒ Andreev bound states $E_{p,k}$

Andreev bound state spectrum is strongly dependent upon symmetry properties of the orbital wavefunction.
Localized states with sub-gap energies form at the junction interface ⇒ Andreev bound states $E_{p,k}$

Andreev bound state spectrum is strongly dependent upon symmetry properties of the orbital wavefunction.

The electronic free energy $F_{el}$ of the junction can be written in terms of these states $E_{p,k}$

$$F_{el} = -2k_B T \int_{|k|=k_F} dk \frac{|k_z|}{k_F} \sum_p \log \left( 2 \cosh \left( \frac{E_{p,k}}{2k_B T} \right) \right)$$

Total free energy is $F_{el} + \frac{|M|^2}{2\chi}$. We numerically minimize with respect to $M$ and $\phi$ to find the stable state of the junction.
A Tale of Two Junctions

\[ \rho_{z\rho_{z}} \text{ Junction} \]

- Always zero energy states

\[ \rho_{y\rho_{y}} \text{ Junction} \]

- Zero energy states only for transparent barrier

\[ \Delta e^{i\phi} \]

\[ \Delta \]

\[ U_{p} \]

\[ \eta \]

\[ d_{L} \]

\[ d_{R} \]
A Tale of Two Junctions

\( \rho_z - \rho_z \) Junction

\( \rho_y - \rho_y \) Junction

\[ M = 0, \ U_P = 0 \]
A Tale of Two Junctions

**$\rho_z-\rho_z$ Junction**

- $d_L$ and $d_R$ denote the normal states of the left and right superconductors, respectively.
- $d_L$ and $d_R$ are connected by a junction barrier.
- The system is described by the order parameter $\eta$.

**$\rho_y-\rho_y$ Junction**

- $d_L$ and $d_R$ denote the normal states of the left and right superconductors, respectively.
- $d_L$ and $d_R$ are connected by a junction barrier.
- The system is described by the order parameter $\eta$.

The diagrams show the boundary conditions for the two junctions at $z < 0$, $z = 0$, and $z > 0$.

**Formulas**

- $M = 0$, $U_p = m/k_F\hbar^2$

---

Philip M. R. Brydon, C. Iniotakis, Dirk Manske, M. Sigrist

Broken time-reversal symmetry...
\[ M = 0.5 g \mu_B m / k_F \hbar^2, \quad U_p = m / k_F \hbar^2 \]
Self-Consistent Solution

\[ U_P = 0.7 m/k_F \hbar^2, \ \eta = 0.2\pi, \ \chi = 20(g\mu_B m/k_F \hbar^2)^2/\Delta_0 \]

- Magnetic instability occurs below temperature \( T_{Curie} < T_c \).
- Occurs over a wide range of \( U_P, \eta \) and \( \chi \) values.
\[ \eta = 0.2\pi, \ \chi = 20(g\mu_B m/k_F\hbar^2)^2/\Delta_0 \]

\[ U_P \leftrightarrow \text{width of barrier, choice of barrier material} \]
(\(M, \phi_m\)) and (\(-M, -\phi_m\)) are degenerate \(\Rightarrow\) domains in barrier

\[
\frac{\phi}{\Phi_0} = n + \oint_C ds \cdot \nabla \phi = n + \frac{\phi_m}{\pi}
\]

fractional flux quanta at domain wall

Critical Current Enhancement

$p_y-p_y$ junction

Large increase in critical current below the magnetic transition: signature of magnetic instability

$\chi = 20/\Delta_0$

$\chi = 10/\Delta_0$

$M=0$

$U_P = 0.7m/k_Bh^2$

$\eta = 0.2\pi$

Philip M. R. Brydon, C. Iniotakis, Dirk Manske, M. Sigrist
time-reversal-symmetry broken by geometric arrangement of triplet superconductors in a Josephson junction
time-reversal-symmetry broken by geometric arrangement of triplet superconductors in a Josephson junction

“effective spin” created at tunneling barrier
Summary

- time-reversal-symmetry broken by geometric arrangement of triplet superconductors in a Josephson junction
- “effective spin” created at tunneling barrier
- coupling to “effective spin” induces a magnetization of the tunneling barrier

Experimental consequences: fractional flux quanta and critical current enhancement

PMRB, Iniotakis, Manske, and Sigrist, cond-mat/0908.2975
PMRB, Manske and Sigrist, JPSJ 77, 103714 (2008); PMRB and Manske, cond-mat/0901.4096; PMRB cond-mat/0908.4065

Thank you for your attention
Summary

- time-reversal-symmetry broken by geometric arrangement of triplet superconductors in a Josephson junction
- “effective spin” created at tunneling barrier
- coupling to “effective spin” induces a magnetization of the tunneling barrier
- functional superconductor interface

Experimental consequences: fractional flux quanta and critical current enhancement

PMRB, Iniotakis, Manske, and Sigrist, cond-mat/0908.2975
PMRB, Manske and Sigrist, JPSJ 77, 103714 (2008);
PMRB and Manske, cond-mat/0901.4096;
PMRB cond-mat/0908.4065

Thank you for your attention
time-reversal-symmetry broken by geometric arrangement of triplet superconductors in a Josephson junction

“effective spin” created at tunneling barrier

coupling to “effective spin” induces a magnetization of the tunneling barrier

functional superconductor interface

Experimental consequences: fractional flux quanta and critical current enhancement
time-reversal-symmetry broken by geometric arrangement of triplet superconductors in a Josephson junction

“effective spin” created at tunneling barrier

coupling to “effective spin” induces a magnetization of the tunneling barrier

functional superconductor interface

Experimental consequences: fractional flux quanta and critical current enhancement

PMRB, Iniotakis, Manske, and Sigrist, cond-mat/0908.2975
PMRB, Manske and Sigrist, JPSJ 77, 103714 (2008); PMRB and Manske, cond-mat/0901.4096; PMRB cond-mat/0908.4065
Summary

- time-reversal-symmetry broken by geometric arrangement of triplet superconductors in a Josephson junction
- “effective spin” created at tunneling barrier
- coupling to “effective spin” induces a magnetization of the tunneling barrier
- functional superconductor interface
- Experimental consequences: fractional flux quanta and critical current enhancement

PMRB, Iniotakis, Manske, and Sigrist, cond-mat/0908.2975
PMRB, Manske and Sigrist, JPSJ 77, 103714 (2008); PMRB and Manske, cond-mat/0901.4096; PMRB cond-mat/0908.4065

Thank you for your attention